放送大学研究年報　第26号（2008）119–125頁
Journal of The Open University of Japan, No. 26（2008）pp.119–125

滑降シンプレックス法による成長曲線からの大気パラメータの決定

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The Determination of Atmospheric Parameters of Curve-of-Growth by the Downhill Simplex Method

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ABSTRACT

We made the program which determine the atmospheric parameters from the curve-of-growth by the downhill simplex method due to Nelder and Mead. This program determines the four variables, Δx, Δy, θ_{ex}, and $\log_{10}2\alpha$ as the best point of the four variables from the starting set of 5 points of the four variables, where Δx is a difference between an empirical curve-of-growth and a theoretical curve-of-growth in the direction parallel to the abscissa and Δy is a difference between an empirical curve-of-growth and a theoretical curve-of-growth in the direction parallel to the ordinate. The objective function is taken to be the variance of lines in curve-of-growth in the direction parallel to the abscissa. This program is the combination between the program of the above downhill simplex method by Sprott and the program of the method 2 for curve-of-growth analysis by Yoshioka.

The effectiveness of this program was tested by comparing the results by this program with that by the program of the method 2 by Yoshioka. The data used for the comparison are those for 86 lines of Fe Ⅰ of HD187203. The following conclusions were drawn from the test.

1) The best point is obtained for the tolerance values of the objective function between 0.001 to 0.00001.
2) The best point depends on the starting set. But, the uncertainty due to the starting set is small in comparison with the one due to other cause in the curve-of-growth analysis.
3) This program is an effective method for the curve-of-growth analysis. In comparison with the program of the method 2 by Yoshioka, this program reaches the four variables in quite short steps and in quite a short time.

要　旨

われわれは、NelderとMeadによる滑降シンプレックス法を適用して成長曲線から大気パラメータを求めるプログラムを作成した。本プログラムは、Δx、Δy、θ_{ex}、and $\log_{10}2\alpha$の４つの変数とする４次元空間中の５個の頂点から成るシンプレックスにおいて、出発点となるシンプレックスから最適の点の座標として４変数を求めるものである。なお、Δxは観測された成長曲線と理論成長曲線の横軸の差を意味し、Δyは両成長曲線の縦軸の差を意味する。ここで、目的関数はプロットされた吸収線の横軸上のちらばりの分散値とした。本プログラムは、Sprottが作成した滑降シンプレックス法のプログラムと吉岡が作成した成長曲線法解析の方法２のプログラムを組み合わせたものである。

われわれは、本プログラムと吉岡の方法２のプログラムの結果を比較することにより、本プログラムの有効性を調べた。比較に使われたデータは、HD187203の86本のFeⅠの吸収線である。そして、次の結論を得た。

1）許される有効数字内での最適の値は、滑降シンプレックス法での許容相対誤差を0.001～0.00001にとれば得られる。
2）最適の値は、出発点となるシンプレックスの選び方によって変わる。しかしその差は、成長曲線法として許される範囲内にある。
3）本プログラムは、吉岡の方法２のプログラムと比べて、かなり短いステップで得られるので、かなり時間を短縮でき、有効な方法と言える。

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Ⅰ. Introduction

A curve-of-growth analysis is one the methods which are used for the analysis of stellar atmospheres. The other method which is mainly used is a model atmosphere analysis. Since detailed distribution of physical quantities such as temperature and pressure and so on are taken into account in a model atmosphere analysis, it is called fine analysis. It is used when accurate observational data are available and the nature of stellar atmosphere is known to a good approximation.

A curve-of-growth analysis is usually used when accurate observational data are not available or there is not enough knowledge about the nature of a stellar atmosphere. In this method, one-layer approximation is made, i. e., it is assumed that there exists a specific value for a physical quantity of the atmosphere such as temperature, pressure and density. A curve-of-growth is used in this method. A curve-of-growth is a graphical representation of the relation between the logarithm of an equivalent width of an absorption line, $\log_{10} W$, and the logarithm of a number density of absorbing atoms, N, times an oscillator strength, f, $\log_{10} Nf$. The equivalent width of a line is the width of the rectangular profile for which the height is equal to the continuum level near the line and the area is equal to that of the line. The equivalent width divided by wavelength of the line, W/λ, is often used instead of W, and some multiplicative factor is often added to Nf. We obtain by this method a representative quantities of atmosphere, for example, electron pressure, gas pressure, ionization temperature, and excitation temperature, T_{ex}, together with chemical composition. This method is also called coarse analysis. Furthermore, it is also called an absolute curve-of-growth analysis or an absolute coarse analysis in order to distinguish it from the analysis mentioned below.

In cases where accurate values for oscillater strength are not known, the values of the abscissa $\log_{10} X_s$ of the curve-of-growth for the standard star for which the physical quantities and the chemical composition of the atmosphere are already known are plotted instead of $\log_{10} Nf$. In this case, the relative values to the standard star for the physical quantities and the chemical composition are obtained instead of the absolute values. This method is called differential curve-of-growth analysis or differential coarse analysis.

Ⅱ. Procedures by Using a Compter Done to Date

The curve-of-growth analysis has conventionally been applied by eye measure. There is a fear that the results obtained by eye measure depend on the subjectivity of an analyzer. Moreover, an objective estimate of error cannot been made by eye measure. The curve-of-growth by using a computer have been applied in order to overcome the above weak points.

For example, Tech (1971)[1] has done a differential curve -of-growth analysis for Ba Ⅱ star ζ Cap, using ε Vir (G8 Ⅲ type star) as a standard star. In this analysis, he determined the differential reciprocal exciataion temperature, $\Delta\theta_{ex}$ ($\theta_{ex}=5040/T_{ex}$), relative to the standard star by the minimum-sigma method, using a computer. Powell (1971)[2] has made computer programs for a differential curve-of-growth analysis of solar-type stars.

Ⅱ-1. The Minimum-Sigma Method by Tech

The minimum-sigma method by Tech (1971)[1] are made in the following way. First a preliminary value for $\Delta\theta_{ex}$ is chosen and the value, $\log_{10} X_{rel}$ is calculated according to the following expression for each line of a given element at the same ionization stage,

$$\log_{10} X_{rel} = \log_{10} X_s - \Delta\theta_{ex}\chi_l, \tag{1}$$

where $\log_{10} X_s$ is the abscissa of a curve-of-growth of a standard star and χ_l is the excitation potential of the lower energy level. Then a mean curve of cubic or quartic polynomial is calculated by the least-squares method to give the best representation of $\log_{10} X_{rel}$ as a function of $\log_{10}(W/\lambda)$, and the standard deviation σ of points from the mean curve in a direction parallel to the $\log_{10} X_{rel}$ axis is calculated. By repeating the above calculation for several values of $\Delta\theta_{ex}$, a correlation between σ and $\Delta\theta_{ex}$ is obtained. A graph of this correlation is generally a smooth curve with a unique minimum. The adopted value of $\Delta\theta_{ex}$ is taken to be that value for which σ is least. Using this value of $\Delta\theta_{ex}$, the empirical curve-of-growth is reconstructed by plotting for each line $\log_{10} X_{rel}$ along the abscissa and $\log_{10}(W/\lambda)$ along the ordinate. Then, this empirical curve-of-growth is fitted to the theoretical curve-of-growth and the horizontal shift of this empirical curve onto the theoretical one gives the quantity which is related to the ratio of the number density of the element at the ionization stage concerned between the star being analyzed and the standard star.

The theoretical curve for ζ Cap was that for pure absorption in a Milne-Eddington atmosphere calculated by Hunger (1956)[3] with damping parameter

$\log_{10}(2\alpha)=-2.5$ and with $\log_{10}(c/2R_cV_D)=4.63$, where c is the speed of light and V_D is the Doppler velocity; R_c is the limiting central depth for strong lines. In the paper by Tech (1971)[1], he wrote, "The theoretical curve that offers the best fit to the majority of the empirical curve-of-growth for ζ Cap, and the one that has been adopted, is that for pure absorption in a Milne-Eddington atmosphere…", but he did not describe the details of the fitting, e. g. the criterion of the best fit.

The strong points of this procedure, which is the reversal of the weak points of the conventional procedure, are as follows: 1) Each line is treated separately and separate weight can be applied to each line; 2) Correct excitation potentials rather than mean values of excitation ranges are taken into account; 3) It gives dispassionately reproducible results and objective estimates of error.

On the other hand, this procedure has the following weak points: 1) Great care must be exercised in assuring that no widely discordant lines are used; 2) Since lines on the flat or the damping portions of the curve-of-growth will dominate the value of σ and mask the variation due to $\Delta\theta_{ex}$, such lines are generally excluded in this analysis, which brings about ambiguity to the results; 3) There is not a guarantee that the mean curve from which the σ values are calculated really represents the distribution of points adequately.

Ⅱ-2. The Procedure Powell

The programs made by Powell (1971)[2] are based on the formulae by Pagel (1964)[4], that is, the abscissa in a curve-of-growth $\log_{10}X$ is normalized so that $\log_{10}X=\log_{10}(W/\lambda)$ for sufficiently weak lines, and for neutral lines, the quantity plotted along the abscissa in an empirical curve-of-growth is not the right side of equation (2) but $\log_{10}X_S+\Delta\theta_{ex}\Delta\chi$, where $\Delta\chi$ is the difference between the ionization potential and the lower excitation potential.

In this procedure, the $\Delta\theta_{ex}$ value and the vertical and horizontal shifts which fit an empirical curve to a theoretical one are first determined, and then the shape of the theoretical curve which fits best to the empirical one, i. e. the damping parameter of the theoretical one is determined.

In the determination of the $\Delta\theta_{ex}$ value and these vertical and horizontal shifts, only the lines which are on the liner portion or on the knee of the flat portion of the curve-of-growth are used, because the $\Delta\theta_{ex}$ value determined from these lines depends only slightly on the shape of the theoretical curve and is not affected very much by the vertical shift adopted in the fitting.

The determination is done in the following iterative way. First, an initial value of $\Delta\theta_{ex}$ is estimated and the empirical curve-of-growth is constructed. Secondly, the empirical curve is fitted to the theoretical one by van der Held (1931)[4] with a damping parameter of $\alpha=0.05$. Van der Held curves-of-growth are those for pure absorption in a Schuster-Schwarzschild atmosphere and Cowley and Cowley (1964)[5] has found that an absolute curve-of-growth for the sun constructed by them fits best to the van der Held curve with $\alpha=0.05$. Thirdly, the theoretical curve fitted to the empirical one is further shifted horizontally in order to normalize it so that it passes through the points of $(-6.5, -6.5)$, and the value of $\log_{10}X$ corresponding to $\log_{10}(W/\lambda)$ for the star being analyzed is read off for each line from this normalized curve. Lastly, a new value of $\Delta\theta_{ex}$ is found from a least-squares solution to the relation,

$$[X]=[A]+\Delta\theta_{ex}\Delta\chi, \tag{2}$$

where square bracket represents the logarithmic difference of the denoted quantity between the star being analyzed and the sun; A is the number ratio of a relevant element and to hydrogen uncorrected for ionization. The above process is repeated until a difference between successive estimate of $\Delta\theta_{ex}$ becomes less than 0.005.

Adopting the values of $\Delta\theta_{ex}$ and of the vertical and horizontal shifts thus determined, the final value of α is determined by obtaining the best fit of the empirical curve to a family of van der Held curves on the condition for a least-squares fit in a direction parallel to the $\log_{10}(W/\lambda)$ axis for all the points in the curve-of-growth. If this value of α is more than a factor of ten greater than or less than 0.05, the above of the determinations of $\Delta\theta_{ex}$ and the shifts is repeated using the new value of α.

In the above process of the determination of $\Delta\theta_{ex}$ values etc.. the fitting of the van der Held curve to the empirical one is done on the assumption that the values for the abscissa are accurately known and the values for the ordinates have a Gaussian error distribution. Consequently, the fitting is done on the condition for a least-squares fit in a direction parallel to the $\log_{10}(W/\lambda)$ axis. This fitting is done in the following way which is also iterative. First, the initial value of the vertical shift Δy_i is taken to be zero and the initial value of the horizontal shift Δx_i is taken to be the mean value of maximum and minimum values of $\log_{10}(W/\lambda)-\log_{10}X_S-\Delta\theta_{ex}\Delta\chi$. Secondly, values of R are calculated for two values $\Delta x_i+0.15$ and of $\Delta x_i-0.15$, where R is the derivative with respect to Δx of of the sum of the squares of the deviation in the ordinate of the empirical curve from the van der Held curve which is shifted horizontally by Δx and shifted vertically by Δy. Thirdly, the Δx value corresponding to $R=0$ is estimated from the two R values and from the above two values by assuming that a linear relation between R and Δx exists. Lastly, using this Δx value instead of Δx_i, a new estimation of the Δx value such that $R=0$ is done. This iteration is continued until the difference between successive estimates of Δx is less than 0.0002. Using the final Δx value, a new value of Δy is estimated as aleast-squares solution in a direction parallel to the ordinate. The whole process is repeated until the difference between successive estimates of Δy is less than 0.00002.

The strong points of this procedure are the same as described for the minimum-sigma method. The weak points of this procedure also are the same as the minimum-sigma method, except for the third point. There is, however, another weak point that there is a fear of divergence in the iterative process.

Ⅱ-3. The Procedure of the Method of Type 2 by Yoshioka

Yoshioka (1987)[6] (Hereafter referred to as Paper Ⅰ) developed a new method. In the new method, the strong points were made use of and the weak points were overcomed. In Paper Ⅰ, two kinds of procedure were developed, which are called the method of type 1 and of type 2 respectively in Paper Ⅰ, and the superiority of the method of type 2 has been indicated.

In the method of type 2, the values of $\Delta\theta_{ex}$ is determined simultaneously with the damping constant and the vertical shift, where the theoretical curve fitted to the empirical curve is that for pure absorption in the Milne-Edington atmosphere calculated by Hunger (1956)[3]. For the Milne-Eddington atmosphere the vertical shift equals to $\log_{10}(c/2R_C V_D)$. The determination is done in the following way. First, the value of damping parameter $\log_{10}2\alpha(2\alpha=\lambda\Gamma/2\pi V_D)$ is settled, where Γ is the damping constant. Secondly, the value of $\Delta\theta_{ex}$ is determined as least-squares solution in the direction parallel to the abscissa for various values of $\log_{10}(c/2R_C V_D)$. Thirdly, the $\log_{10}(c/2R_C V_D)$ value and the corresponding value of $\Delta\theta_{ex}$ which give a minimum value of the standard deviation σ_{temp} of the $\Delta\theta_{ex}$ value are selected. The above process is repeated for various values of $\log_{10}2\alpha$, and the $\log_{10}2\alpha$ and the corresponding values of $\Delta\theta_{ex}$ and $\log_{10}(c/2R_C V_D)$ for which the σ_{temp} value is minimum are adopted as the final values for these quantities. In the above process, a gradient of the theoretical curve-of-growth for the ordinate of a line is taken into account as a weight for a least-squares solution so that the lines on the linear and damping parts of the curve-of-growth are given heavier weight than those on the flat part of the curve-of-growth.

The strong points of this procedure are the same as described for the minimum-sigma method by Tech (1971)[1] and for the procedure by Powell (1971)[2]. The weak points of both of the procedures, i. e., the ambiguity in the use of lines and the inconsistency in the use of curve-of-growth are overcomed in this procedure, for this procedure uses all the lines and the same curve-of-growth are used for the determination of $\log_{10}2\alpha$, V_D, and$\Delta\theta_{ex}$.

Ⅲ. New Procedure by Using the Downhill Simplex Method

In this paper, we made the program which simplifies this procedure. We reports the approach and the result of this program.

The procedures above described can be regarded as one of optimization problems where the optimal solution is the set of four variables, Δx, Δy, $\Delta\theta_{ex}$, and $\log_{10}2\alpha$. The objective function in our problem which is minimized by the optimal solution can be selected according the criterion of agreement between empirical curve-of-growth and theoretical one. For example, the variance of lines in curve-of-growth in the direction parallel to the ordinate is selected as the objective function in the the procedure by Powell (1971)[2]. On the other hand, the variance of lines in curve-of-growth in the direction parallel to the abscissa is selected as the objective function in method of type 2 by Yoshioka (1987)[6].

We made the program which solves this optimization problem by the downhill simplex method due to Nelder and Mead[7](hereafter referred to as DSM). DSM is the method which finds approximately the minimum of an objective function of more than one independent variables without constraints. DMS requires only function evaluations and it does not require derivatives. DMS is widely used in physical sciences and in engineerings.

Ⅲ-1. The Process of the Downhill Simplex Method

In DSM, a simplex is the geometrical figure consisting, in N dimensions (or the number of independent variables), of $N+1$ points (or vertices) and all their interconnecting line segments, polygonal faces. In DSM, the determination of solution is done in the following iterative way.

[1] It starts with $N+1$ points, where these points are indicated as x_i(i=1, 2, …, $N+1$), defining an initial simplex.

[2] These points are put in order of ascending values of $f(x_i)$ where f is the objective function, and the fol-

lowing inequality holds,

$f(x_1) \leqq f(x_2) \leqq \cdots \leqq f(x_{N+1})$. (3)

[3] The centroid of points $\langle x \rangle$ of the simplex without x_{N+1} is calculated by the following expression,

$\langle x \rangle = \Sigma_1^n X_i / n$. (4)

Then, the reflection point x_r is calculated by the following expression,

$x_r = \langle x \rangle + \rho(\langle x \rangle - x_{N+1})$, (5)

where ρ is taken to be 1 in this study.

[4] The point x_{N+1} is replaced by the point x_r in the case where the following inequality holds,

$f(x_1) \leqq f(x_r) \leqq f(x_N)$. (6)

The new set of points are put in order of ascending values of $f(x_i)$ as is done in the step [2]. Then, we go to the terminating step [13].

When the inequality (6) does not holds, we go to the step [5].

[5] We go to the step [6] in the case where the following inequality holds,

$f(x_r) < f(x_1)$. (7)

When the following inequality holds,

$f(x_N) < f(x_r) < f(x_{N+1})$, (8)

we go to the step [8].

When the following inequality holds,

$f(x_{N+1}) < f(x_r)$, (9)

we go to the step [10].

[6] The expansion point x_e is calculated by the following expression,

$x_e = \langle x \rangle + \chi(\langle x \rangle - x_{N+1})$, (9)

where χ is taken to be 2 in this study.

[7] The point x_{N+1} is replaced by the point x_e in the case where the following inequality holds,

$f(x_e) < f(x_r)$. (10)

The new set of points are put in order of ascending values of $f(x_i)$ as is done in the step [2]. Then, we go to the terminating step [13].

When the inequality (10) does not holds, the point x_{N+1} is replaced by the point x_r, and the new set of points are put in order of ascending values of $f(x_i)$. Then, we go to the terminating step [13].

[8] The outside contraction point x_{oc} is calculated by the following expression,

$x_{oc} = \langle x \rangle + \gamma(x_r - \langle x \rangle)$, (11)

where γ is taken to be 0.5 in this study.

[9] The point x_{N+1} is replaced by the point x_r in the case where the following inequality holds,

$f(x_{oc}) < f(x_{N+1})$. (12)

Then, the point x_{N+1} is replaced by the point x_{oc} and the new set of points are put in order of ascending values of $f(x_i)$. Then, we go to the terminating step [13].

When the inequality (12) does not holds, we go to the step [12].

[10] The inside contraction point x_{ic} is calculated by the following expression,

$x_{ic} = \langle x \rangle + \gamma(x_{N+1} - \langle x \rangle)$, (13)

where γ is taken to be 0.5 in this study.

[11] The point x_{N+1} is replaced by the point x_{ic} in the case where the following inequality holds,

$f(x_{ic}) < f(x_{N+1})$. (14)

The new set of points are put in order of ascending values of $f(x_i)$. Then, we go to the terminating step [13].

When the inequality (14) does not holds, we go to the step [12].

[12] All the points x_i except for the point x_1 are replaced by the shrink points x_{si} which are calculated by the following expression,

$x_{si} = x_1 + \sigma(x_i - x_1)$, (15)

where σ is taken to be 0.5 in this study. The new set of points are put in order of ascending values of $f(x_i)$. Then, we go back to the step [3].

[13] The program terminates in the case where the relative fractional range from the highest value $f(x_{N+1})$ to the lowest value $f(x_1)$ is smaller than some tolerance *ftol*, i. e., in the case where the following inequality holds,

$ftol < 2|f(x_{N+1}) - f(x_1)| / |f(x_{N+1}) + f(x_1)|$. (16)

The program pauses in the case where the number of iteration *ITER* exceeds some tolerance *ITMAX*, i. e., in the case where the following inequality holds,

$ITER > ITMAX$. (17)

In the cases where neither the inequality (16) nor the inequality (17) does not hold, we go to the step [3].

In the above process of DSM, the minimum value of $f(x)$ and the corresponding values of variables of N dimensions are determined as the value of $f(x_1)$ and the values of the point x_1 at the terminating step [13], respectively.

Ⅲ-2. The Results by Using Our Program

We made the program which selects the best set of four variables, Δx, Δy, $\Delta\theta_{ex}$, and $\log_{10} 2\alpha$. The determination of the best set is done by DSM of 4 dimensions where the objective function is the variance of lines in curve-of-growth in the direction parallel to the abscissa as it is selected in method of type 2 by Yoshioka(1987)[6]. Our program is the combination between the program of DSM by Sprott[7] and the program by Yoshioka(1987)[6]. The latter program calculates the value of the objective function for a given set of four variables, Δx, Δy, $\Delta\theta_{ex}$, and $\log_{10} 2\alpha$.

We have tested the effectiveness of our program by comparing the result by our program with that by the program by Yoshioka(1987)[6]. The data used for the comparison are that for Fe Ⅰ lines of HD187203 which is a supergiant with F8 type. The number of the Fe I lines is equal to 86. An absolute curve-of-growth analysis is done for the above data by the program by Yoshioka (1987)[6] and the following set of four variables, Δx, Δy, θ_{ex} (instead of $\Delta\theta_{ex}$ in the case of an absolute curve-of-growth analysis) and $\log_{10}2\alpha$ is obtained; $\Delta x=-3.075$, $\Delta y=4.63$, $\theta_{ex}=1.02$, and $\log_{10}2\alpha=-1.85$. The value of the objective function f is equal to 2.435970256.

We have obtained the best sets of four variables by our program, altering the starting set of 5 points of a simplex and the *ftol* value. The ranges of the starting values which were taken in this dtudy are as follows, $\Delta x=-3.40\sim-2.60$; $\Delta y=4.30\sim5.10$; $\theta_{ex}=0.60\sim1.40$; $\log_{10}2\alpha=-1.40\sim-2.20$. These ranges were chosen so that they include the above values obtained by the program by Yoshioka (1987)[6]. The *ftol* values taken in this study are $0.1\sim1\times10^{-9}$.

For example, for the following starting set ①, $\Delta x_i=-2.90-0.05i$, $\Delta y_i=4.80-0.05i$, $\theta_{exi}=1.15-0.05i$, and $\log_{10}2\alpha_i=-2.00+0.05i$, where $i=1, \cdots, 5$, and for $ftol=1\times10^{-9}$, the following best point is obtained; $\Delta x=-3.034$, $\Delta y=4.67$, $\theta_{ex}=1.02$, and $\log_{10}2\alpha=-1.87$. The corresponding f and *ITER* values are equal to 2.455285751 and 63, respectively.

On the other hand, for the following starting set ②, $\Delta x_i=-2.40-0.20i$, $\Delta y_i=5.30-0.20i$, $\theta_{exi}=1.60-0.20i$, and $\log_{10}2\alpha_i=-2.40+0.20i$, and for $ftol=1\times10^{-9}$, the following best point is obtained; $\Delta x=-2.988$, $\Delta y=4.71$, $\theta_{ex}=1.01$, and $\log_{10}2\alpha=-1.81$. The corresponding f and *ITER* value are equal to 2.646665165 and 65.

As is exemplified above, the best point depends not only on the *ITER* value but also on the starting set. The starting set ③ that gives the best point with the smallest f value are as follows; $\Delta x_i=-2.90-0.05i$, $\Delta y_i=4.80-0.05i$, $\theta_{exi}=1.60-0.20i$, and $\log_{10}2\alpha_i=-2.40+0.20i$, and the best point and corresponding f and *ITER* value are as follows; $\Delta x=-3.046$, $\Delta y=4.65$, $\theta_{ex}=1.02$, and $\log_{10}2\alpha=-1.82$, 2.441517739 and 71. This set is nealy equal to that obtained by the program by Yoshioka (1987)[6].

The best point depend also on the combination of the starting point even when it is equal as a set. For example, for the following starting set ④, $\Delta x_i=-2.40-0.20i$, $\Delta y_i=5.30-0.20i$, $\theta_{exi}=1.60-0.20i$, and $\log_{10}2\alpha_i=-1.20-0.20i$, the following best point is obtained; $\Delta x=-2.988$, $\Delta y=4.71$, $\theta_{ex}=1.01$, and $\log_{10}2\alpha=-1.79$. The corresponding f and *ITER* value are equal to 2.642037475 and 70. On the other hand, for the following starting set ⑤, $\Delta x_i=-2.40-0.20i$, $\Delta y_i=4.10+0.20i$, $\theta_{exi}=1.60-0.20i$, and $\log_{10}2\alpha_i=-1.20-0.20i$, the following best point is obtained; $\Delta x=-2.989$, $\Delta y=4.69$, $\theta_{ex}=1.01$, and $\log_{10}2\alpha=-1.79$. The corresponding f and *ITER* value are equal to 2.542638882 and 71. The starting sets ②, ④, and ⑤ are equal as a set, but the best set differ slightly.

Ⅳ. Conclusions and Discussion

The following conclusions are drawn from the above calculation.

1) The best points given above are the result for $ftol=1\times10^{-9}$, but the same results are already obtained for the *ftol* values between $0.001\sim0.00001$ in the case where the significant figure of the values of Δy, θ_{ex}, and $\log_{10}2\alpha$ is two and the significant figure for the Δx value is three.
2) The best point depends on the starting set. According to the starting set, the best points of Δx, Δy, θ_{ex}, and $\log_{10}2\alpha$ differs by ±0.05, ±0.06, ±0.01, and ±0.09, respectively. The θ_{ex} value is insensitive to the best set, and the Δx value is relatively insensitive to the best set. The Δy and $\log_{10}2\alpha$ values are more sensitive to the starting set than the Δx and θ_{ex} values. However, these uncertainties are small in the curve-of-growth analysis. The uncertainty of ±0.01 in θ_{ex} value is small, and the uncertainty of $\pm$ 0.05 in abundance in logarithmic scale which originates from that in Δx value is also small. The uncertainty of ±0.06 in Doppler broadening in logarithmic scale which originates from that in Δy value is small. The uncertainty of ±0.09 in $\log_{10}2\alpha$ value is also small.
3) There are some starting sets which does not converge to the best set. For example, for the following starting set, $\Delta x_i=-2.40-0.20i$, $\Delta y_i=4.80-0.05i$, $\theta_{exi}=1.60-0.20i$, and $\log_{10}2\alpha_i=-2.00+0.05i$, it does not converge in the case where the *ftol* value is smaller than 0.00007. However, in these cases, the f value converges the same value that the starting set converges for the *ftol* value which is larger by more than a factor of ten than that which does not give a convergent set.
4) In conclusion from the above description, our program is an effective method for the curve-of-growth. In comparison with the program by Yoshioka (1987)[6], our program reaches the Δx, Δy, $\Delta\theta_{ex}$, and $\log_{10}2\alpha$ values in quite short steps and in quite short time with acceptable accuracy.

The following problems are left for the future.

1) The variance of lines in curve-of-growth in the direction parallel to the abscissa is selected as the objective function in this study. It is interesting that this study is done in the case where other variations are selected as the objective function.

2) It is interesting that DMS is applied not only in the curve-of-growth method but also in fine analysis.
3) It may be effective to apply the method such as the simulated annealing method in order to avoid converging in the local minimum before converging in a global minimum as in our case.

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(2008年11月3日受理)